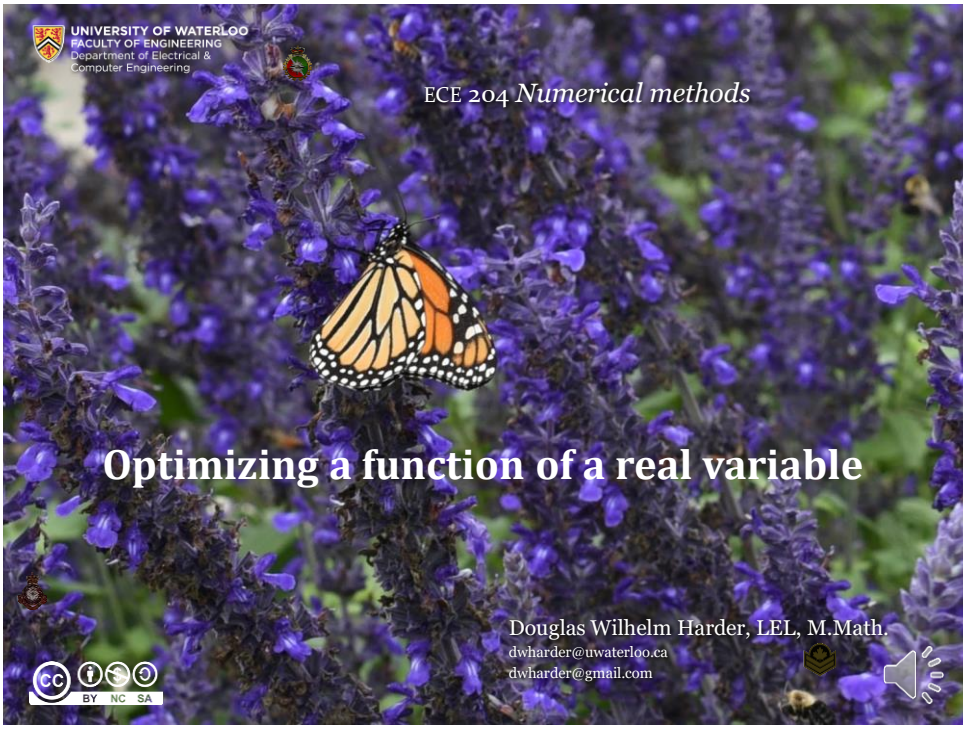
UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical & Computer Engineering
ECE 204 Numerical methods
Optimizing a function of a real variable
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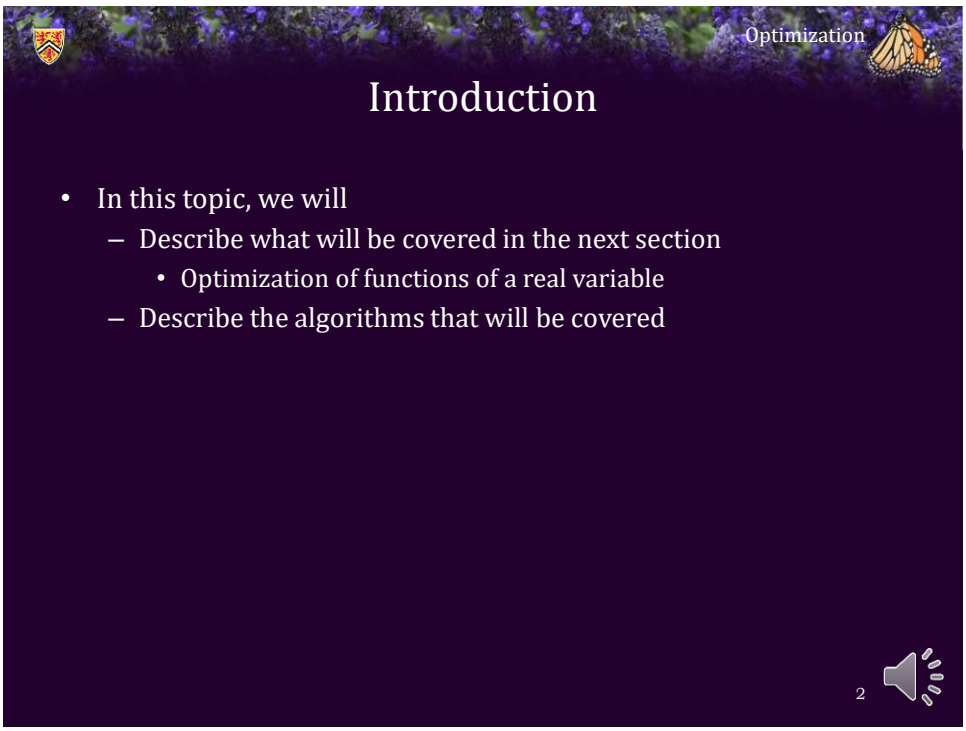

Optimization
Introduction
In this topic, we will
Describe what will be covered in the next section
Optimization of functions of a real variable
Describe the algorithms that will be covered
2

2

The problem

- For a differentiable function of one variable,
  it is much easier to define an extreme point:
  - The derivative is zero
  - The first non-zero derivative after that is an even derivative
- If you are looking for a minimum,
  you can always follow the gradient
  - At a point x_0,
    - If $f^{(1)}(x_0) > 0$, a minimum may exist to the left
    - Otherwise, if $f^{(1)}(x_0) < 0$, a minimum may exist to the right

Issues with optimization

- If a function is real and differentiable,
  then the Taylor series no longer has the second term:

$$f(x_{\min} + h) = f(x_{\min}) + \frac{1}{2} f^{(2)}(\xi) h^2$$

Looking ahead

- We will look at:
  - A step-by-step algorithm that walks towards the minimum
  - Newton's method applied to the derivative
  - Golden-section search
  - Successive parabolic interpolation
  - The Brent-Dekker optimization method
- The relationships are as follows:
  - The first will parallel the bisection method
  - The second parallel's Newton's method
  - The third is an optimization for the first
  - The fourth uses interpolating quadratic polynomials
  - The last is a hybrid of previous algorithms

5

Summary

- Following this topic, you now
  - Have an overview of the ideas to be covered in this section
  - Understand that we will look at five algorithms for optimizing a real-valued function of a real variable

6

References

[1] https://en.wikipedia.org/wiki/Mathematical_optimization

7

Acknowledgments

None so far.

8

Colophon

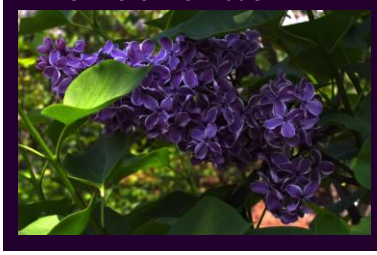
These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using `Consolas`. Mathematical equations are prepared in MathType by Design Science, Inc.

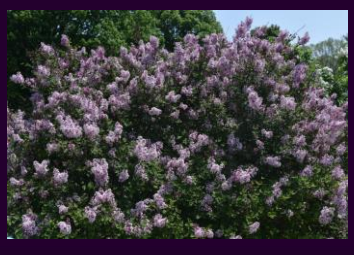
Examples may be formulated and checked using Maple by Maplesoft, Inc.

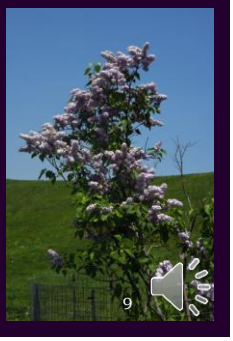
The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

https://www.rbg.ca/

for more information.







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